EXERCISE - BINOMIAL THEOREM

LEVEL – I – SUBJECTIVE QUESTIONS

1. Expand the following expressions and find the number of term in the expansion of the expressions.
(a) $(2x+3y)^{99}$ **(b)** $\left(1+2\sqrt{3}a\right)^{19}+\left(1-2\sqrt{3}a\right)^{19}$
(c) $\left(\sqrt{p}+\sqrt{q}\right)^{12}-\left(\sqrt{p}-\sqrt{q}\right)^{12}$ **(d)** $(1+x+x^2)^n$
(e) $(1-x+x^2)^n$ **(f)** $(ax-y)^{10}-(ax+y)^{10}$
(g) $(1+2x+x^2)^{27}$

2. Write all the terms of the expansion of the following expression using binomial theorem.
(a) $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4, x\neq 0$ **(b)** $(3x^2-2ax+3a^2)^3$
(c) $\left(a+\frac{1}{b}\right)^4$ **(d)** $\left(x+\frac{1}{x}\right)^9$

3. Expand $(x+y)^5-(x-y)^5$ and find the value of $\left(\sqrt{3}+1\right)^5-\left(\sqrt{3}-1\right)^5$.

4. If O is the sum of odd terms and E is the sum of even term in the expansion of $(x+a)^n$, then prove that;
(a) $O^2-E^2=(x^2-a^2)^n$
(b) $4OE=(x+a)^{2n}-(x-a)^{2n}$
(c) $2(O^2+E^2)=(x+a)^{2n}+(x-a)^{2n}$

5. Prove that $\sum_{r=1}^{n} {}^nC_r\, 3^r=4^n$.

6. Evaluate the following : **(a)** $(99)^5$ **(b)** $(102)^6$
(c) $(10.1)^5$ **(d)** $(96)^3$ **(e)** $(0.99)^5+(1.01)^5$
(f) $\left(\sqrt{2}+\sqrt{3}\right)^7+\left(\sqrt{2}-\sqrt{3}\right)^7$
(g) $\left(a^2+\sqrt{a^2-1}\right)^5+\left(a^2+\sqrt{a^2-1}\right)^5$

7. State binomial approximation and approximation of $(0.99)^5$ using the first three terms of its expansion.

8. Which is larger **(a)** $(1.01)^{1000000}$ or 10000
(b) $(1.1)^{10000}$ or 1000 **(c)** $(1.2)^{4000}$ or 800.

9. Prove that $(101)^{50}>(100)^{50}+(99)^{50}$.

10. Using binomial theorem, prove that 6^n-5n always leaves remainder 1 when divided by 25.

11. Using binomial theorem, prove that $2^{3n}-7n-1$ is divisible by 49, where $n\in N$.

12. Show that $3^{2n+2}-8n-9$ is divisible by 64, whenever n is a positive integer.

13. If n is a positive integer, show that $3^{3n}-26n-1$ is divisible by 676.

14. Write the general term in the expansion of **(a)** $(x^2-y)^6$, **(b)** $(x^2-xy)^{99}, x\neq 0$.

15. Write the 11^{th} term in the following expansion:
(a) $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}, x\neq 0$ **(b)** $\left(\frac{e^x+e^{-x}}{2}\right)^n$.

16. Find the middle term(s) in the following expansions:
(a) $\left(x-\frac{1}{x}\right)^{10}$ **(b)** $\left(3-\frac{x^3}{7}\right)^7$ **(c)** $\left(2a+\frac{b}{3}\right)^{12}$
(d) $(1-2x+x^2)^n$ **(e)** $(1+3x+3x^2+x^3)^{2n}$

17. Find the middle term(s) in the expansion of $\left(\frac{e^{ix}+e^{-ix}}{2}\right)^n$ when **(a)** n is odd **(b)** n is even.

18. Find the 11^{th} term from the end in the expansion of $\left(x^{3/2}y^{1/2}-x^{1/2}y^{3/2}\right)^{23}$.

19. Find the coefficient of a^5b^7 in the expansion of $\left(a-\sqrt{3}b\right)^{12}$.

20. In the expansion of $(x+y)^n$ coefficients of seventh and thirteenth terms are equal. Find the value of n.

21. If the coefficients of $(2r+4)^{th}$ term and $(r+2)^{th}$ term in the expansion of $(1+x)^{43}$ are equal, find r.

22. In the expansion of $(a+b)^n$ coefficients of a^{n-12} and b^{n-11} are equal. Find the value of n.

23. In the expansion of $(1+a)^{m+n}$, prove that the coefficients of a^m and a^n are equal.

24. Prove that the term independent of a in the expansion of $\left(a+\frac{1}{a}\right)^{2n}$ is $\frac{1.3.5\ldots(2n-1)}{n!}2^n$.

25. Find the number of real negative term in the expansion of $(1-ix)^{4n-2}, n\in N$ and $x>0$.

26. If in the expansion of $(1+x)^n$, the coefficients of p^{th} and q^{th} term are equal, prove that $p+q=n+2$, where $p\neq q$.

LEVEL – I – OBJECTIVE QUESTIONS

27. The coefficient of x^{-17} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$ is
(A) 1365 (B) −1365 (C) 495 (D) −495

28. The coefficient of $\frac{1}{x}$ in the binomial expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$ is
(A) $\frac{n!}{(n-1)!(n+1)!}$ (B) $\frac{(2n)!}{(n-1)!(n+1)!}$
(C) $\frac{(2n)!}{(2n-1)!(2n+1)!}$ (D) N.O.T.

29. If r^{th} term in the expansion of $\left(2x^2-\frac{1}{x}\right)^{12}$ is without x, then r is equal to
(A) 8 (B) 7 (C) 9 (D) 10

30. In the expansion of $\left(x^2-\frac{1}{3x}\right)^9$, the term without x is equal to
(A) $\frac{28}{81}$ (B) $-\frac{28}{243}$ (C) $\frac{28}{243}$ (D) $-\frac{28}{81}$

31. In the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$, x^{-17} is r^{th} term, then $r=$

(A) 10 (B) 11 (C) 12 (D) 13

32. In the expansion of $\left(\frac{1}{2}x^{1/3}+x^{-5}\right)^8$ the term independent of x is (A) 5^{th} term (B) 6^{th} term (C) 7^{th} term (D) 8^{th} term

33. The total number of term in the expansion of $(x+a)^{100}+(x-a)^{100}$ after simplification is,
(A) 202 (B) 51 (C) 50 (D) 101

34. If the coefficient of x in $\left(x^2+\frac{\lambda}{x}\right)^5$ is 270, then $\lambda=$
(A) 3 (B) 4 (C) 5 (D) 8

35. The coefficient of term independent of x in the expansion of $\left(ax+\frac{b}{x}\right)^{16}$ is
(C) $16!\,a^8b^8$ (B) $\frac{16!}{8!}a^8b^8$ $\frac{16!}{(8!)^2}a^8b^8$ (D) $\frac{16!}{(8!)^3}a^8b^8$

36. The term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ will be
(A) 3/2 (B) 5/4 (C) 5/2 (D) N.O.T.

37. If in the expansion of $(a+b)^n$ and $(a+b)^{n+3}$, the ratio of the coefficients of the second and third terms, and third and fourth terms respectively are equal, then the value of n is
(A) 3 (B) 4 (C) 5 (D) 6

38. If A and B are the sums of odd and even terms respectively in the expansion of $(x+a)^n$, then $(x+a)^{2n}-(x-a)^{2n}$ is equal to
(A) $4(A+B)$ (B) $4(A-B)$ (C) AB (D) $4AB$

39. If A and B are the sums of odd and even terms respectively in the expansion of $(x+a)^n$, then $(x+a)^n(x-a)^n$ is equal to
(A) A^2-B^2 (B) A^2+B^2 (C) AB (D) $4AB$

40. Two middle terms in the expansion of $\left(x-\frac{1}{x}\right)^{11}$ are
(A) $231x$ and $\frac{231}{x}$ (B) $462x$ and $\frac{462}{x}$
(C) $-462x$ and $-\frac{462}{x}$ (D) N.O.T.

41. The largest term in the expansion of $(3+2x)^{50}$ where $x=1/2$ is
(A) 5^{th} (B) 51^{st} (C) 7^{th} (D) 6^{th}

42. The value of the natural numbers n such that the inequality $2^n>2n+1$ is valid is
(A) For all $n\geq 3$ (B) For all $n<3$
(C) For all values of n (D) N.O.T.

43. $\frac{C_1}{C_0}+2\frac{C_2}{C_1}+3\frac{C_3}{C_2}+\cdots+15\frac{C_{15}}{C_{14}}=$
(A) 100 (B) 120 (C) -120 (D) N.O.T.

44. If the coefficient of x in the expansion of $\left(x+\frac{k}{x}\right)^5$ is 270, then $k=$
(A) 1 (B) 2 (C) 3 (D) 4

45. The sum of the coefficients in the expansion of $(x+y)^n$ is 4096. The greatest coefficient in the expansion is
(A) 1024 (B) 924 (C) 824 (D) 724

46. 2^{60} when divided by 7 leaves the remainder
(A) 1 (B) 6 (C) 5 (D) 2

47. If T_2/T_3 in the expansion of $(x+y)^2$ and T_3/T_4 in the expansion of $(x+y)^{n+3}$ are equal, then $n=$
(A) 3 (B) 4 (C) 5 (D) 6

48. The coefficient of middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and $(1-\alpha x)^6$ is same if α equals **[AIEEE 2004]**
(A) 3/5 (B) 10/3 (C) 3/10 (D) $-3/10$

LEVEL – II – SUBJECTIVE QUESTIONS

49. Find the term independent of x in the expansion of $\left(\sqrt[3]{x}+\frac{1}{2\sqrt[3]{x}}\right)^{18}, x>0$.

50. Find the coefficient of x^{10} in the binomial expansion of $\left(2x^2-\frac{3}{x}\right)^{11}, x\neq 0$.

51. Find a positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6.

52. Which term in the expansion of $\left\{\left(\frac{x}{\sqrt[2]{y}}\right)^{1/3}+\left(\frac{y}{\sqrt[3]{x}}\right)^{1/2}\right\}^{11}$ contains x and y to one and the same power.

53. Prove that the coefficient of $(r+1)^{th}$ term in the expansion of $(1+x)^{1+n}$ is equal to the sum of the coefficients of the r^{th} and $(r+1)^{th}$ terms in the expansion of $(1+x)^n$.

54. Prove that the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is sum of the coefficients of the middle terms in the expansion of $(1+x)^{2n-1}$ and is equal to $\frac{1.3.5\ldots(2n-1)}{n!}2^n$.

55. The sum of the coefficients of the first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^m, x\neq 0$, m being a natural number is 559. Find the term of the expansion containing x^3.

56. If the coefficients of $(r-5)^{th}$ and $(2r-1)^{th}$ terms in the expansion of $(1+x)^{34}$ are equal, find r.

57. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$.

58. If the fourth term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is 5/2, then find the value of a and n.

59. Find the coefficient of x^5 in the product $(1+2x)^6(1-x)^7$ using binomial theorem.

60. If the third term in the expansion of $\left(\frac{1}{x}+x^{\log_{10}x}\right)^5$ is 1000. Find the value of x.

61. If the fourth term in the expansion of $\left(\frac{1}{x}+x^{\log_{10}x}\right)^{11}$ is 1650. Find the value of x.

62. In the expansion of $\left(\sqrt{x^{\frac{1}{\log x+1}}}+x^{\frac{1}{12}}\right)^6$, the fourth term is 200 and $x>1$, then find x.

63. For what value of x is the ninth term in the expansion of $\left(3^{\log_3\sqrt{25^{x-1}+7}}+3^{(-1/8)\log_3(5^{x-1}+1)}\right)^{10}$ is equal to 180.

64. Find the value of $\lambda, \lambda \neq 0$ for which the coefficients of the middle term in the expansion of $(1+\lambda x)^4$ and $(1-\lambda x)^6$ are equal.

65. If the coefficients of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$ are equal, find the relation between a and b.

66. Find a, if the coefficient of x^2 and x^3 are equal in the expansion of $(3+ax)^9$.

67. If the coefficient of $2^{nd}, 3^{rd}$ and 4^{th} terms in the expansion of $(1+x)^{2n}$ are in A.P., then prove that $2n^2-9n+7=0$.

68. If the coefficients of a^{r-1}, a^r and a^{r+1} in the expansion of $(1+a)^n$ are in arithmetic progression, prove that $n^2-n(4r+1)+4r^2-2=0$.

69. Show that the coefficient of α^{50} in the product of $(1+\alpha)^{41}(1-\alpha+\alpha^2)^{40}$ is zero.

70. Find the coefficient of x^5 in the expansion of $(1+x)^{31}+(1+x)^{32}+\cdots+(1+x)^{40}$.

71. In the expansion of $(1+x)^n$ the binomial coefficients of three consecutive terms are respectively 220, 495 and 792, find the value of n and the position of the terms of these coefficients.

72. Find the greatest value of the term independent of x in expansion of $\left(x\cos\theta+\frac{\cos\theta}{x}\right)^{10}$, where $\theta \in R$.

73. If in the expansion of $(1-x)^{2n-1}$, the coefficient of x^r are denoted by a_r, then show that $a_{r-1}+a_{2n-r}=0$.

74. In the binomial expansion of $(1+x)^n$, the coefficients of ninth, tenth and eleventh terms are in A.P. Find all the values of n.

75. The coefficients of the $(r-1)^{th}, r^{th}$ and $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio 1: 3: 5. Find n and r.

76. If a_1, a_2, a_3, a_4 be the coefficients of four consecutive terms in the expansion of $(1+x)^n$, then prove that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$.

77. If the $3^{rd}, 4^{th}$ and 5^{th} terms in the expansion of $(x+a)^n$ are respectively 84, 280 and 560. Find the value of x, a and n.

78. How many terms are free from radical signs in the expansion of $\left(x^{1/5}+y^{1/10}\right)^{55}$.

79. Find the number of integral terms in the expansion of $\left(5^{1/2}+7^{1/8}\right)^{1028}$.

80. If the binomial coefficient of $2^{nd}, 3^{rd}, 4^{th}$ and 5^{th} terms in the expansion of $(\alpha+\beta)^n$ be respectively a, b, c and d, then prove that $\frac{b^2-ac}{c^2-bd}=\frac{2a}{c}$.

81. If the binomial coefficient of 11^{th}, 12^{th}, 13^{th}, and 14^{th} terms in the expansion of $(\alpha+\beta)^n$ be respectively a, b, c, and d then, prove that $\frac{b^2-ac}{c^2-bd}=\frac{13a}{11c}$.

82. If a and b are the coefficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then find a as a function of b.

83. If a and b are the coefficients of x^m and x^n in the expansion of $(1+x)^m.(1+x)^n$ respectively, then find the relation between a and b.

84. Prove that the greatest term in the expansion of $(1+x)^{2n}$ has also the greatest coefficient, then $x \in \left(\frac{n}{n+1}, \frac{n+1}{n}\right)$.

85. Find the last two digits of 3^{400}.

LEVEL – II – OBJECTIVE QUESTIONS

86. If the coefficient of x^2 and x^3 in the expansion of $(3+ax)^9$ are the same, then a is,
(A) $-7/9$ (B) $-9/7$ (C) $9/7$ (D) $9/7$

87. The number of terms with integral coefficients in the expansion of $\left(17^{1/3}+35^{1/2}\right)^{600}$ is
(A) 100 (B) 50 (C) 150 (D) 101

88. The number of irrational terms in the expansion of $\left(4^{1/5}+7^{1/10}\right)^{45}$ is
(A) 40 (B) 5 (C) 41 (D) N.O.T.

89. If $(1+x)^{15}=C_0+C_1x+C_2x^2+\ldots\ldots+C_{15}x^{15}$, then $C_2+2C_3+3C_4+\ldots\ldots+14C_{15}=$
(A) 14.2^{14} (B) $13.2^{14}+1$ (C) $13.2^{14}-1$ (D) N.O.T.

90. The unit digit of $17^{1983}+11^{1983}-7^{1983}$ is,
(A) 1 (B) 2 (C) 3 (D) 0

91. If $x+\frac{1}{x}=1$ and $x^{4000}+\frac{1}{x^{4000}}=\lambda$ and μ be the digit at unit place in the number $2^{2^n}+1, n \in N$, and $n>1$ then $\lambda+\mu$ is equal to
(A) 8 (B) 7 (C) 6 (D) 5

92. In the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$, coefficient of x^4 is
(A) 405/256 (B) 504/259 (C) 450/263 (D) N.O.T.

93. If $\sum_{r=0}^{n}\left(\frac{r+2}{r+1}\right).{}^nC_r=\frac{2^n-1}{6}$, then n is equal to
(A) 6 (B) 3 (C) 8 (D) 5

94. Number of terms in the expansion of $\left(\frac{x^3+1+x^6}{x^3}\right)^{\sum n}$, where $n \in N$ is,
(A) $\sum n+1$ (B) ${}^{\Sigma n++2}C_2$ (C) $2n+1$ (D) n^2+n+1

95. For a positive integer n, let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{2^n - 1}$. Then,
(A) $a(100) \le 100$ (B) $a(100) > 100$
(C) $a(200) \le 100$ (D) N.O.T.

96. If $(1 + ax)^n = 1 + 8x + 24x^2 + \cdots$, then the value of a and n is
(A) 2,4 (B) 2,3 (C) 3,6 (D) 1,2

97. $(115)^{96} - (96)^{115}$ is divisible by,
(A) 15 (B) 17 (C) 19 (D) 21

98. If $n \in N$, then $7^{2n} + 2^{3n-3}.3^{n-1}$ is always divisible by
(A) 25 (B) 35 (C) 45 (D) N.O.T.

99. If 5^{40} is divided by 11, then remainder is α and when 2^{2003} is divided by 17, then remainder is β. Then the value of $\beta - \alpha$ is,
(A) 3 (B) 5 (C) 7 (D) 8

100. The value of $\frac{18^3+7^3+3.18.7.25}{3^6+6.243.2+15.81.4+20.27.8+15.9.16+6.3.32+64}$ is
(A) 1 (B) 5 (C) 25 (D) 100

101. The range of the values of the term independent of x in the expansion of $(x \sin^{-1}\theta + \cos^{-1}\theta/x)^{10}$, $\theta \in [-1,1]$ is
(A) $\left[\frac{^{10}C_5 \pi^{10}}{2^5}, -\frac{^{10}C_5 \pi^{10}}{2^{20}}\right]$ (B) $\left[-\frac{^{10}C_5 \pi^{10}}{2^5}, \frac{^{10}C_5 \pi^{10}}{2^{20}}\right]$
(C) $\left[\frac{^{10}C_5 \pi^5}{2^5}, \frac{^{10}C_5 \pi^5}{2^{20}}\right]$ (D) $\left[-\frac{^{10}C_5 \pi^5}{2^5}, \frac{^{10}C_5 \pi^5}{2^{20}}\right]$

102. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$, then the value of $C_0 + 2C_1 + 3C_2 + \cdots + (n+1)C_n =$
(A) $(n+2)2^{n-1}$ (B) $(n+1)2^n$
(C) $(n+1)2^{n-1}$ (D) $(n+2)2^n$

103. If $(1+x)^n = \sum_{r=0}^{n} a_r x^r$ and $b_r = 1 + \frac{a_r}{a_{r-1}}$ and $\prod_{r=1}^{n} b_r = \frac{(101)^{100}}{100!}$, then the value of n is,
(A) 99 (B) 100 (C) 101 (D) 102

104. The value of x in the expression $\left(\frac{1}{x} + x^{\log_{10} x}\right)^5$, if the third term in the expansion is 10^6,
(A) 10 (B) 11 (C) 12 (D) N.O.T.

105. If n is an integer greater than 1, then $a - {}^nC_1(a-1) + {}^nC_2(a-2) + \cdots + (-1)^n(a-n)$ is equal to
(A) a (B) 0 (C) a^2 (D) 2^n

106. The term independent of t in the expansion of $\left(t^{-1/6} - t^{1/3}\right)^9$ is
(A) 84 (B) 8.4 (C) 0.84 (D) −84

107. The sum of $1 + n\left(1 - \frac{1}{x}\right) + \frac{n(n+1)}{2!}\left(1 - \frac{1}{x}\right)^2 + \cdots \infty$ will be
(A) x^n (B) x^{-n} (C) $\left(1 - \frac{1}{x}\right)^n$ (D) N.O.T.

108. If $\frac{(1-3x)^{1/2} + (1-x)^{5/3}}{\sqrt{4-x}}$ is approximately equal to $a + bx$ for small values of x, then (a, b)=
(A) $\left(1, \frac{35}{24}\right)$ (B) $\left(1, -\frac{35}{24}\right)$ (C) $\left(2, \frac{35}{12}\right)$ (D) $\left(2, -\frac{35}{12}\right)$

109. Let $P(n)$ denote the statement that $n^2 + n$ is odd. It is seen that $P(n) \Rightarrow P(n+1)$, P_n is true for all
(A) $n > 2$
(B) $n > m$, m being a fixed positive integer
(C) Nothing can be said (D) N.O.T.

110. The least remainder when 17^{30} is divided by 5 is,
(A) 1 (B) 2 (C) 3 (D) 4

111. If $n \in N$, then $11^{n+2} + 12^{2n+1}$ is divisible by
(A) 113 (B) 123 (C) 133 (D) N.O.T.

112. If the sum of the coefficients in the expansion of $(\alpha^2x^2 - 2\alpha x + 1)^{51}$ vanishes, then value of α is
(A) 2 (B) −1 (C) 1 (D) −2

113. In the polynomial $(x-1)(x-2)(x-3)\ldots(x-100)$ the coefficient of x^{99} is
(A) 5050 (B) −5050 (C) 100 (D) 99

114. The sum of the coefficients of the polynomial $(1 + x - 3x^2)^{2163}$ is
(A) 0 (B) 1 (C) −1 (D) 2^{2163}

115. If C_r stands for ${}^nC_r$ and $n > 0$, then the value of $\sum_{k=0}^{n} k^3 \left(\frac{C_k}{C_{k-1}}\right)^2$ is
(A) $\frac{n(n+1)(n+2)}{12}$ (B) $\frac{n(n+1)^2}{12}$
(C) $\frac{n(n+1)(n+2)^2}{12}$ (D) N.O.T.

116. The sum of the series $1 + \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + \cdots$ is equal to
(A) $1/\sqrt{5}$ (B) $1/\sqrt{2}$ (C) $\sqrt{5/3}$ (D) $\sqrt{5}$

117. If the value of x is so small that x^2 and greater powers can be neglected, then $\frac{\sqrt{1+x} + \sqrt[3]{(1-x)^2}}{1 + x + \sqrt{1+x}}$ is equal to
(A) $1 + \frac{5}{6}x$ (B) $1 - \frac{5}{6}x$ (C) $1 + \frac{2}{3}x$ (D) $1 - \frac{2}{3}x$

118. The coefficient of x^{100} in the expansion of $\sum_{r=0}^{200} (1+x)^r$ is
(A) $\binom{200}{100}$ (B) $\binom{201}{102}$ (C) $\binom{200}{101}$ (D) $\binom{201}{100}$

119. If 6^{th} term in the expansion of the binomial $\left[\sqrt{2^{\log(10-3^x)}} + \sqrt[5]{2^{(x-2)\log 3}}\right]^n$ is equal to 21 and it is known that the binomial coefficients of the 2^{nd}, 3^{rd} and 4^{th} terms in the expansion represent respectively the first, third and fifth terms of an A.P. (the symbol log stands for logarithm to the base 10), then $x =$
(A) 0 (B) 1 (C) 2 (D) N.O.T.

120. In the binomial expansion of $(a - b)^n$, $n \ge 5$, the sum of the 5^{th} and 6^{th} terms is zero. Then a/b is equal to **[IIT-JEE 2000]**
(A) $\frac{1}{6}(n-5)$ (B) $\frac{1}{5}(n-4)$ (C) $\frac{5}{n-4}$ (D) $\frac{6}{n-5}$

121. If $(1 - x + x^2)^n = a_0 + a_1x + a_2x^2 + \cdots a_{2n}x^{2n}$, then $a_0 + a_2 + a_4 + \cdots + a_{2n} =$
(A) $\frac{3^n+1}{2}$ (B) $\frac{3^n-1}{2}$ (C) $\frac{1-3^n}{2}$ (D) $3^n + \frac{1}{2}$

122. If $(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$, then find the value of

$\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\dots\left(1+\frac{C_n}{C_{n-1}}\right)=$
(A) $\frac{n^{n-1}}{(n-1)!}$ (B) $\frac{(n+1)^{n-1}}{(n-1)!}$ (C) $\frac{(n+1)^n}{n!}$ (D) $\frac{(n+1)^{n+1}}{n!}$

123. For any natural number m and n, if $(1-t)^m(1+t)^n = 1 + a_1y + a_2y^2 + \cdots$, and $a_1 = a_2 = 10$, then (m,n) is,
(A) (35,20) (B) (45,35) (C) (35,45) (D) (20,45)

124. The number of rational term in the expansion of $\left(1+\sqrt{2}+\sqrt[3]{3}\right)^6$ is,
(A) 5 (B) 6 (C) 7 (D) 8

125. The number of irrational terms in the expansion of $\left(\sqrt[8]{5}+\sqrt[6]{2}\right)^{100}$ is
(A) 97 (B) 98 (C) 96 (D) 99

126. The value of the expression $\{{}^{n+1}C_2 + [{}^2C_2 + {}^3C_2 + {}^4C_2 + \cdots + {}^nC_2]\}$ is
(A) $\sum n$ (B) $\sum n^2$ (C) $\sum n^3$ (D) $(n+1)/2$

127. The coefficient of x^{50} in the expansion of $(1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \cdots + 1001x^{1000}$ is
(A) ${}^{1000}C_{50}$ (B) ${}^{1001}C_{50}$
(C) ${}^{1002}C_{50}$ (D) ${}^{1000}C_{51}$

128. The value of $\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \cdots$ is
(A) $\frac{2^n}{n+1}$ (B) $\frac{2^n-1}{n+1}$ (C) $\frac{2^n+1}{n+1}$ (D) N.O.T.

129. The value of x, for which the 6^{th} term of the expansion $\left\{2^{\log_2\sqrt{9^{x-1}+7}} + \frac{1}{2^{(1/5)\log_2(3^{x-1}+1)}}\right\}^7$ is 84, is equal to
(A) 4 (B) 3 (C) 2 (D) 1

ENGINEERING ENTERANCE EXAM QUESTIONS

OBJECTIVE QUESTIONS

130. The coefficient of x^n in the expansion of $(1+x)(1-x)^n$ is **[AIEEE 2004]**
(A) $(n-1)$ (B) $(-1)^n(1-n)$
(C) $(-1)^{n-1}(n-1)^2$ (D) $(-1)^{n-1}n$

131. Unit digit of $3^{1001}.7^{1003}.13^{1003}$ is **[AIEEE 2007]**
(A) 1 (B) 3 (C) 7 (D) 9

132. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is, **[AIEEE 2009]**
(A) 2 (B) 7 (C) 8 (D) 0

133. The term independent of x in the expansion of $\left\{\frac{x+1}{x^{2/3}-x^{-1/3}+1} - \frac{x-1}{x-x^{1/2}}\right\}^{10}$ is, **[JEE Mains 2013]**
(A) 120 (B) 210 (C) 310 (D) 4

134. Given positive integers $r > 1$, $n > 2$ and the coefficient of the $(3r)^{th}$ and $(r+2)^{th}$ terms in the binomial expansion of $(1+x)^{2n}$ are equal, then **[IIT-JEE 1980]**
(A) $n = 2r$ (B) $n = 3r$ (C) $n = 2r+1$ (D) N.O.T.

135. The expression $\left[x+(x^3-1)^{1/2}\right]^5 + \left[x-(x^3-1)^{1/2}\right]^5$ is a degree of polynomial **[IIT-JEE 1992]**
(A) 5 (B) 6 (C) 7 (D) 8

136. If C_r stands for ${}^nC_r$, then the sum of the given series $\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}[C_0^2 - 2C_1^2 + 3C_2^2 - \cdots + (-1)^n(n+1)C_n^2]$, where n is an even positive integer, is equal to **[IIT-JEE 1986]**
(A) $(-1)^{n/2}(n+2)$ (B) $(-1)^n(n+1)$
(C) $(-1)^{/2n}(n+2)$ (D) 0

137. If in the expansion of $(1+x)^m(1-x)^m$, the coefficient of x and x^2 are 3 and – 6 respectively, then m is **[IIT-JEE 1999]**
(A) 6 (B) 9 (C) 12 (D) 24

138. The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, $\left(\text{where } \binom{p}{q} = 0 \text{ if } p < q\right)$ is maximum when m is, **[IIT-JEE 2002]**
(A) 5 (B) 10 (C) 15 (D) 20

139. The coefficient of t^{24} in the expansion of $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is **[IIT-JEE 2003]**
(A) $2+{}^{12}C_6$ (B) ${}^{12}C_5$ (C) ${}^{12}C_6$ (D) $1+{}^{12}C_6$

140. If ${}^{n-1}C_r = (k^2-3).{}^nC_{r+1}$, then k belongs to **[IIT-JEE 2004]**
(A) $(-\infty,-2]$ (B) $[2,\infty)$ (C) $\left[-\sqrt{3},\sqrt{3}\right]$ (D) $(\sqrt{3},2]$

141. $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} - \cdots + \binom{30}{20}\binom{30}{30}$ is equal to **[IIT-JEE 2005]**
(A) ${}^{30}C_{11}$ (B) ${}^{60}C_{10}$ (C) ${}^{30}C_{10}$ (D) ${}^{65}C_{55}$

142. For r =0, 1, 2,… 10; let A_r, B_r and C_r denotes respectively the coefficient of x_r in the expansions of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}C_r)$ is equal to **[IIT-JEE 2010]**
(A) $B_{10} - C_{10}$ (B) $A_{10}(B_{10}^2 - C_{10}A_{10})$
(C) 0 (D) $C_{10} - B_{10}$

ANALYTICAL AND DISCRIPTIVE QUESTIONS

143. Prove that $({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - \cdots + ({}^{2n}C_{2n})^2 = (-1)^n.{}^{2n}C_n$ **[IIT-JEE 1978]**

144. Prove that $C_1^2 - 2C_2^2 + 3C_3^2 - \cdots - 2n.C_{2n}^2 = (-1)^n n.C_n$. **[IIT-JEE 1979]**

145. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$, then, show that the sum of the products of the C_i's taken two at a time represented by $\sum\sum C_iC_j$, $(0 \le i \le j \le n)$ is equal to $2^{2n-1} - \frac{(2n)!}{2(n!)^2}$. **[IIT-JEE 1983]**

146. Given, $s_n = 1 + q + q^2 + q^3 + \cdots + q^n$ and $S_n = 1 + \frac{(q+1)}{2} + \left(\frac{q+1}{2}\right)^2 + \left(\frac{q+1}{2}\right)^3 + \cdots + \left(\frac{q+1}{2}\right)^n$, $q \ne 1$. Prove that

$({}^{n+1}C_1 + {}^{n+1}C_2 s_1 + {}^{n+1}C_3 s_2 + \cdots + {}^{n+1}C_{n+1} s_n) = 2^n S_n$ **[IIT-JEE 1984]**

147. If $\sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r (x-3)^r$ and $a_k = 1$ for all $k \geq n$, then show that $b_n = {}^{2n+1}C_{n+1}$. **[IIT-JEE 1992]**

148. Prove that $\sum_{r=1}^{k} (-3)^{r-1} . {}^{3n}C_{2r-1} = 0$, where $k = 3n/2$ and n is an even positive integer. **[IIT-JEE 1993]**

149. Let n be a positive integer and $(1+x+x^2)^n = a_0 + a_1 x + a_2 x^2 + \cdots + a_{2n} x^{2n}$. Show that $a_0^2 - a_2^2 + \cdots + a_{2n}^2 = a_n$. **[IIT-JEE 1994]**

150. Find the sum of rational terms in the expansion of $\left(\sqrt{2} + 3^{1/5}\right)^{10}$. **[IIT-JEE 1997]**

151. Prove that $\frac{3!}{2(n+3)} = \sum_{r=0}^{n} (-1)^r ({}^nC_r / {}^{r+3}C_r)$. **[IIT-JEE 1997C]**

152. For any positive integer m, n (with $m \geq n$), let $\binom{n}{m} = {}^nC_m$. Prove that $\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \cdots + \binom{m}{m} = \binom{n+1}{m+1}$. Hence or otherwise, prove that $\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + \cdots + (n-m+1)\binom{m}{m} = \binom{n+2}{m+2}$. **[IIT-JEE 2000]**

153. Prove that $2^k \binom{n}{0}\binom{n}{k} - 2^{k-1}\binom{n}{1}\binom{n-1}{k-1} + 2^{k-2}\binom{n}{2}\binom{n-2}{k-2} - \cdots + (-1)^k \binom{n}{k}\binom{n-k}{0} = \binom{n}{k}$. **[IIT-JEE 2003]**

ASSERTION AND REASON

This section contains reasoning type question and has 4 choices, A, B, C and D, out of which only ONE is correct.

(A) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation of Statement–1.
(B) Statement–1 is True, Statement–2 is True; Statement–2 is NOT a correct explanation of Statement–1.
(C) Statement–1 is True, Statement–2 is False;
(D) Statement–1 is False, Statement–2 is True

154. Statement–1: $\sum_{r=0}^{n} (r+1) . {}^nC_r = (n+2)2^{n-1}$ **[AIEEE 2008]**
Statement–2: $\sum_{r=0}^{n} (r+1) . {}^nC_r x^r = (1+x)^n + nx(1+x)^{n-1}$

155. Let $S_1 = \sum_{j=0}^{10} \{j(j-1) . {}^{10}C_j\}$, $S_2 = \sum_{j=0}^{10} (j . {}^{10}C_j)$ and $S_3 = \sum_{j}^{10} (j^2 . {}^{10}C_j)$ **[AIEEE 2010]**
Statement–1: $S_3 = 55 \times 2^9$
Statement–2: $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$

LEVEL – III – FOOD FOR THOUGHT

156. If $\{x\}$ denotes the fractional part of x, then $\left\{\frac{3^{2n}}{8}\right\}$, $n \in N$, is equal to
(A) 3/8 (B) 7/8 (C) 1/8 (D) N.O.T.

157. If S be the sum of coefficients in the expansion of $(\alpha x + \beta y - \gamma z)^n$, where $(\alpha, \beta, \gamma) > 0$, then the value of $\lim_{n \to \infty} \frac{S}{[S^{1/n} + 1]}$ is
(A) $e^{\left(\frac{\alpha\beta}{\gamma}\right)}$ (B) $e^{\left(\frac{\alpha+\beta-\gamma}{\alpha+\beta-\gamma+1}\right)}$ (C) $\frac{\alpha\beta}{\gamma}$ (D) 0

158. The number of rational terms in the expansion of $\left(2^{1/2} + 2^{1/3} + 5^{1/6}\right)^{10}$ is
(A) 1 (B) 2 (C) 3 (D) 4

159. Let $R = \left(5\sqrt{5} + 11\right)^{2n+1}$ and $f = R - [R]$, where $[.]$ denotes greatest integer function. The value of $R.f$ is
(A) 4^{2n+1} (B) 4^{2n} (C) 4^{2n-1} (D) 4^{-2n}

160. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n$; then $C_1 - 3C_3 + 5C_5 - \cdots$ is equal to
(A) $n . 2^{\left(\frac{n-1}{2}\right)} . \cos\left[\left(\frac{n-1}{4}\right)\pi\right]$
(B) $n . 2^{\left(\frac{n-1}{2}\right)} . \sin\left[\left(\frac{n-1}{4}\right)\pi\right]$
(C) $(n-1) . 2^{\left(\frac{n}{2}\right)} . \cos\left[\left(\frac{n-1}{4}\right)\pi\right]$ (D) N.O.T.

161. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$ for every value of θ, then
(A) $b_0 = 1, b_1 = 3$ (B) $b_0 = 0, b_1 = n^2 - 3n + 3$
(C) $b_0 = -1, b_1 = n$ (D) $b_0 = 0, b_1 = n$

162. The sum of the series $\sum_{r=0}^{n} (-1)^r . {}^nC_r \left(\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} + \cdots m \text{ terms}\right)$ is
(A) $\frac{2^{mn}-1}{2^{mn}(2^n-1)}$ (B) $\frac{2^{mn}-1}{2^n-1}$ (C) $\frac{2^{mn}+1}{2^n+1}$ (D) N.O.T.

163. $\binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + \cdots + 2^n\binom{n}{n}$ is equal to
(A) 2^n (B) 0 (C) 3^n (D) N.O.T.

164. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n$, then $\sum\sum_{0 \leq i \leq j \leq n} (C_i + C_j)^2$ equals
(A) $n . {}^{2n}C_n + 2^{2n}$ (B) $(n-1) . {}^{2n}C_n + 2^{2n}$
(C) $(n+1) . {}^{2n}C_n + 2^{2n}$ (D) N.O.T.

165. The coefficient of x^n in the expansion of $(1 - 2x + 3x^2 - 4x^3 + \cdots \text{to } \infty)^{-n}$ is
(A) $\frac{(2n)!}{n!(n-1)!}$ (B) $\frac{(2n)!}{[(n-1)!]^2}$ (C) $\frac{(2n)!}{(n!)^2}$ (D) N.O.T.

166. The value of a and b so that coefficient x^n in the expansion of $\frac{a+bx}{(1-x)^2}$ is $2n+1$, are
(A) 1,1 (B) 1,2 (C) 2,3 (D) N.O.T.

167. Let n and k be positive integers such that $n \geq \frac{k(k+1)}{2}$. The number of solutions $(x_1, x_2 \ldots . x_k)$, $x_1 \geq 1, x_2 \geq 2, \ldots, x_k \geq k$, all integers, satisfying $x_1 + x_2 + \cdots + x_k = n$, is
(A) ${}^mC_{k-1}$ (B) ${}^mC_{k+1}$ (C) ${}^mC_k$ (D) N.O.T.
$\left\{Where\ m = \frac{1}{2}(2n - k^2 + k - 2)\right\}$

168. If the sum of the coefficients in the expansion of $(ax^2-2x+1)^{35}$ is equal to the sum of the coefficients in the expansion of $(x-ay)^{35}$, then $a=$
(A) 0 (B) May be any real number
(C) 1 (D) No such value exist

169. When P is a natural number, then $P^{n+1}+(P+1)^{2n-1}$ is divisible by
(A) P (B) P^2+P (C) P^2+P+1 (D) P^2-1

170. Algebraically greatest term in the expansion of $(3-5x)^{11}$, when $x=1/5$ is,
(A) 55×3^8 (B) 55×3^9 (C) 110×3^9 (D)N.O.T.

171. To expand $(1+2x)^{-1/2}$ as an infinite series, the range of x should be
(A) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (B) $\left(-\frac{1}{2},\frac{1}{2}\right)$ (C) $[-2,2]$ (D) $(-2,2)$

172. Given $S_{n+1}=3S_n-2S_{n-1}$ and $S_0=2, S_1=3$, then the value of S_n for all $n\in N$ is,
(A 2^n-1 (B) 2^n+1 (C) 0 (D) N.O.T.

173. If a,b,c,d be four consecutive coefficients in the binomial expansion of $(1+x)^n$, then the value of the expression $\left\{\left(\frac{b}{b+c}\right)^2-\frac{ac}{(a+b)(c+d)}\right\}$, where $x>0$, is
(A) <0 (B) >0 (C) $=0$ (D) 2

174. The coefficients of x^2y^2, yzt^2 and $xyzt$ in the expansion of $(x+y+z+t)^4$ are in ratio,
(A) $4:2:1$ (B) $1:2:4$ (C) $2:4:1$ (D) $1:4:2$

175. If the expansion of $\frac{1}{(1-ax)(1-bx)}$ is $a_0+a_1x+a_2x^2+\cdots$, then a_n is
(A) $\frac{a^n-b^n}{b-a}$ (B) $\frac{a^{n+1}-b^{n+1}}{b-a}$
(C) $\frac{b^{n+1}-a^{n+1}}{b-a}$ (D) $\frac{b^n-a^n}{b-a}$

COMPREHENSION

176. PASSAGE: *Let n be a positive integer, such that* $(1+x+x^2)^n=a_0+a_1x+a_2x^2+\cdots+a_{2n}x^{2n}$, *then*

(I) The value of a_r is, for (all $r\in[0,2n]$) (A) a_{2n-r} (B) a_{n-r} (C) a_{2n} (D) $n.a_{2n-1}$

(II) The value of $a_0+a_1+a_2+\cdots+a_{n-1}$ is (A) $\frac{1}{2}3^n$ (B) $\frac{1}{2}(3^n-a_n)$ (C) $\frac{1}{2}a_n$ (D) $3.a_n^n$

(III) The value of $a_0^2-a_1^2+a_2^2-a_3^2+\cdots+a_{2n}^2$ is (A) a_n (B) $\left(\frac{n+1}{2}\right)\sum_{i=0}^{2n}a_i$ (C) $2a_n$ (D) $3a_n$

177. PASSAGE: *Numerically greatest term in the expansion of* $(x+a)^n$.
Let T_{r+1} *be* r^{th} *term in the binomial expansion of* $(x+a)^n$ *and* T_{r+1} *be the numerically greatest term. Then,* $\left|\frac{T_r}{T_{r+1}}\right|\ge 1$ *and also* $\left|\frac{T_{r+1}}{T_{r+2}}\right|\le 1$. *In the binomial expansion* $T_r={}^nC_{r-1}x^{n-r+1}a^{r-1}$, $T_{r+1}={}^nC_rx^{n-r}a^r$ *and* $T_{r+2}={}^nC_{r+1}x^{n-r-1}a^{r+1}$.
So $\left|\frac{T_r}{T_{r+1}}\right|=\left(\frac{n-r+1}{r}\right)\left|\frac{a}{x}\right|\le 1\Rightarrow r\le\frac{n+1}{(x/a)+1}$ $(=k$ *say*$)$.
If k is an integer, then both T_k *and* T_{k+1} *are numerically greatest term.*
If k is not an integer, then $T_{[k]}$ *is numerically greatest term, where* $[k]$ *denotes the greatest integer function.*

(I) Numerically greatest term in the expansion of $(1-2x)^8$, when $x=2$ is
(A) 7.2^{14} (B) 35.2^9 (C) 2^{17} (D) N.O.T.

(II) Magnitude wise the greatest term in the expansion of $(3-2x)^9$, when $x=1$ is
(A) ${}^9C_23^72^2$ (B) ${}^9C_33^62^3$ (C) ${}^9C_43^52^4$ (D) both B and C

(III) Given that 4^{th} term in the expansion of $\left(2+\frac{3}{8}x\right)^{10}$ has the maximum numerical value, the range of value of x is
(A) $x\in\left[-\frac{64}{21},-2\right]\cup\left[2,\frac{64}{21}\right]$ (B) $-\frac{64}{21}<x<-2$ (C) $\frac{64}{21}<x<4$ (D) N.O.T.

(IV) If n is even positive integer, then the condition that the numerically greatest term in the expansion of $(1+x)^n$ may also have the greatest coefficient is
(A) $\frac{n}{n+2}<|x|<\frac{n+2}{n}$ (B) $\frac{n+1}{n}<|x|<\frac{n}{n+1}$ (C) $\frac{n}{n+4}<|x|<\frac{n+4}{n}$ (D) $\frac{n}{n+3}<|x|<\frac{n+3}{n}$

(V) The interval in which x $(x>0)$ lies so that numerically greatest term also have the greatest coefficient in the expansion of $(1-x)^{21}$ is,
(A) $\left[\frac{5}{6},\frac{6}{5}\right]$ (B) $\left(\frac{5}{6},\frac{6}{5}\right)$ (C) $\left(\frac{4}{5},\frac{5}{4}\right)$ (D) $\left[\frac{4}{5},\frac{5}{4}\right]$

178. PASSAGE: *If* α *and* β *are positive integers and* t *is a positive integer, which is not a perfect square, then the number* $\alpha+\beta\sqrt{t}$ *and its positive integral powers are essentially irrational. If n is a positive integer, then it is noticed that* $E=\left(\alpha+\beta\sqrt{t}\right)^n+\left(\alpha-\beta\sqrt{t}\right)^n$, *where* $E\in I^+$. *Further if* $0<\left(\alpha-\beta\sqrt{t}\right)^n<1$ *then E is the integer just next to* $\left(\alpha+\beta\sqrt{t}\right)^n$. *From the equality* $\left(\alpha+\beta\sqrt{t}\right)^n+\left(\alpha-\beta\sqrt{t}\right)^n=E$, *it can be deduced that sum of fractional parts is equal to 1. Similarly,* $\left(\alpha+\beta\sqrt{t}\right)^n-\left(\alpha-\beta\sqrt{t}\right)^n$ *is also a positive integer, where* $0<\left(\alpha-\beta\sqrt{t}\right)^n<1$.

(I) If $R = (5\sqrt{5}+11)^{2n+1}$ and $f = R - [R]$, then Rf must be equal to
(A) 5^{2n+1} (B) 11^{2n+1} (C) 4^{2n+1} (D) 1

(II) Let $S_n = (\sqrt{3}+1)^{2n} + (\sqrt{3}-1)^{2n}$, then
(A) $4S_{n+1} = 2S_n - 3S_{n-1}$ (B) $S_{n+1} = 4S_n - S_{n-1}$ (C) $S_{n+1} = 8S_n - 4S_{n-1}$ (D) $S_{n+1} = 2S_n$

(III) The integer just greater than $(\sqrt{3}+1)^{2n}$ is always divisible by,
(A) 2^{n+3} (B) 2^{n+2} (C) 2^{n+1} (D) 2^{n+4}

179. PASSAGE: *If $C_0, C_1, C_2, \ldots, C_n$ are binomial coefficients, then $(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$. Various relations among binomial coefficients can be derived by putting $x = 1, -1$, $x = i$, $x = \omega$, $(i = \sqrt{-1}$, $\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i)$. Some other identities can be derived by adding and subtracting two identities. The expression $(a+ib)^n$ can be evaluated by using de Moivre's identity by putting $a = r\cos\theta$ and $b = r\sin\theta$.*

(I) The value of the expression $({}^nC_0 - {}^nC_2 + {}^nC_4 - {}^nC_6 + \cdots)^2 + ({}^nC_1 - {}^nC_3 + {}^nC_5 - \cdots)^2$ must be
(A) 2^{2n} (B) 2^n (C) 2^{n^2} (D) N.O.T.

(II) The value of ${}^nC_0 + {}^nC_4 + {}^nC_8 + \cdots$ is
(A) $2^{n/2}\cos\frac{n\pi}{8}$ (B) $2^{n/2}\sin\frac{n\pi}{8}$ (C) $2^n + 2^{n/2}\cos\frac{n\pi}{4}$ (D) $2^{n-2} + 2^{\frac{n-2}{2}}\cos\frac{n\pi}{4}$

(III) The value of $\sum_{r=1}^{k}(-3)^{r-1}\,{}^{3n}C_{2r-1}$, where n is an even positive integer and $k = \frac{3n}{2}$ is
(A) 3^n (B) 3^{n-1} (C) 6^n (D) 0

LEVEL – III – MULTIPLE CORRECT CHOICE QUESTIONS

180. Which of the following is correct?
(A) $(101)^{100} > (100)^{101}$ (B) $(26)^{25} > (25)^{26}$
(C) $(300)^{299} < (299)^{300}$ (D) $(70)^{71} = (71)^{70}$

181. In the expansion of $\left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$
(A) the number of irrational terms = 19
(B) middle term is irrational
(C) the number of rational term = 2
(D) 9^{th} term is rational

182. If $C_0, C_2, C_3, \ldots, C_n$ have their usual meanings, then $\frac{C_0}{n(n+1)} - \frac{C_1}{(n+1)(n+2)} + \frac{C_2}{(n+2)(n+3)} + \cdots$ to $(n+1)$ terms is equal to
(A) $\int_0^1 x^{n+1}(1-x)^{n+1}\,dx$
(B) $\int_0^1 x^n(1+x)^{n+1}\,dx$
(C) $\int_0^1 x^{n-1}(1-x)^{n+1}\,dx$
(D) $\int_0^1 x^{n+1}(1-x)^{n-1}\,dx$

183. The series $\frac{{}^nC_0}{n} + \frac{{}^nC_1}{n+1} + \frac{{}^nC_2}{n+2} + \cdots + \frac{{}^nC_n}{2n}$ equal to
(A) $\int_0^1 x^{n-1}(1-x)^n\,dx$ (B) $\int_0^1 x^{n-1}(1+x)^n\,dx$
(C) $\int_0^1 x^{n-1}(1-x)^n\,dx$ (D) $\int_0^1 x^n(x-1)^{n-1}\,dx$

184. If the sum of the coefficients in the expansion of $(2+3cx+c^2x^2)^{12}$ vanishes then c equals
(A) -2 (B) 2 (C) 1 (D) -1

185. If $f = (2+\sqrt{3})^n - [(2+\sqrt{3})^n]$, where [ ] is greatest integer function $\le x$, then $\frac{f^2}{1-f}$ is
(A) a positive integer (B) a negative integer
(C) an rational number (D) N.O.T.

186. Let $R = (8+3\sqrt{7})^{20}$ and $[R]$ = the greatest integer less than or equal to R, then
(A) $[R]$ is even (B) $[R]$ is odd
(B) $R - [R] = 1 - \frac{1}{(8+3\sqrt{7})^{20}}$ (D) $R + [R]R = 1 + R^2$

187. The value of ${}^nC_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \cdots + {}^{n+k}C_k$ is equal to,
(A) ${}^{n+k+1}C_k$ (B) ${}^{n+k+1}C_{n+1}$
(C) ${}^{n+k}C_{n+1}$ (D) N.O.T.

188. Number of values of r satisfying the equation ${}^{69}C_{3r-1} - {}^{69}C_{r^2} = {}^{69}C_{r^2-1} - {}^{69}C_{3r}$ is,
(A) 1 (B) 2 (C) 3 (D) 7

189. If $(1+2x+3x^2)^{10} = a_0 + a_1x + a_2x^2 + \cdots + a_{20}x^{20}$, then
(A) $a_1 = 20$ (B) $a_2 = 210$
(C) $a_4 = 8085$ (D) $a_{20} = 2^2.3^7.7$

190. If $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + \cdots + a_{2n}x^{2n}$, then the value of $a_0 + a_3 + a_6 + \cdots$ is
(A) $a_1 + a_4 + a_7 + \cdots$ (B) $a_2 + a_5 + a_8 + \cdots$
(C) 3^{n-1} (D) N.O.T.

191. The largest coefficient in the expansion of $(4+3x)^{35}$ is,
(A) ${}^{25}C_{11}3^{25}\left(\frac{4}{3}\right)^{14}$ (B) ${}^{25}C_{11}4^{25}\left(\frac{3}{4}\right)^{11}$
(C) ${}^{25}C_{14}4^{14}3^{11}$ (D) ${}^{25}C_{11}4^{11}3^{14}$

192. If $ac > b^2$, then the sum of the coefficients in the expansion of $(a\alpha^2x^2 + 2b\alpha x + c)^n$ is, where a, b, c, $\alpha \in R$ and $n \in R$
(A) positive if $a > 0$ (B) positive if $c > 0$

(C) negative if $a < 0$ and n is odd
(D) positive if $c < 0$ and n is even

193. If n is a positive integer, then in the trinomial expansion of $(x^2 + 2x + 2)^n$, the coefficient of
(A) x is $2^n.n$ (B) x^2 is $n^2.2^n - 1$
(C) x^3 is $2^n.{}^{n+1}C_3$ (D) All of the above

194. Suppose $x_1, x_2, x_3, \ldots, x_n$ (n>2) are real numbers, such that $x_i = -x_{n-i+1}$ for $1 \le i \le n$. Consider the sum $S = \sum\sum\sum x_i x_j x_k$, $0 \le i, j, k \le n$ and i, j, k are distinct. Then, which of the following is not true?
(A) $S = n!\, x_1.x_2.x_3 \ldots x_n$ (B) $S = (n-3)(n-4)$
(C) $S = (n-3)(n-4)(n-5)$ (D) $S = 0, \forall\, n \in N$

195. If n is a positive integer and $\left(3\sqrt{3}+5\right)^{2n+1} = P + f$, where $P \in I$ and 0<f<1, then
(A) P is an even integer
(B) $(P+f)^2$ is divisible by 2^{2n+1}
(C) the integer just below $\left(3\sqrt{3}+5\right)^{2n+1}$ is divisible by 3
(D) P is divisible by 10

196. The number of distinct term of the expansion of $(x + 2y + 3z - 5u + 7w)^n$ is
(A) n+1 (B) ${}^{n+4}C_4$
(C) ${}^{n+4}C_n$ (D) $\frac{(n+1)(n+2)(n+3)(n+4)}{24}$

197. The coefficient of $a^8 b^6 c^4$ in the expansion of $(a+b+c)^{18}$ is
(A) ${}^{18}C_{14}.{}^{14}C_8$ (B) ${}^{18}C_{10}.{}^{10}C_6$
(C) ${}^{18}C_6.{}^{12}C_8$ (D) ${}^{18}C_4.{}^{14}C_6$

LEVEL – III – SUBJECTIVE QUESTIONS

198. Find a, b and n in the expansion of $(a+b)^n$ if the first three terms of the expansion are 729, 7290 and 30375 respectively.

199. If a and b are distinct integers, prove that $a - b$ is a factor of $a^n - b^n$, whenever n is a positive integer.

200. Prove that the coefficients of x^m in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2n}$ is $\frac{(2n)!}{\left(\frac{4n-m}{3}\right)!\left(\frac{2n+m}{3}\right)!}$.

201. Does the expansion $\left(2x^2 - \frac{1}{x}\right)^{20}$ contain any term involving x^9.

202. Prove that there is no term involving x^6 in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{11}$.

203. In the expansion of $(1+x)^n$, for all n<50, find all the values of n for which coefficients of three consecutive terms are in A.P. Also find consecutive terms for the corresponding values of n.

204. If the coefficient of $(r+1)^{th}$, $(r+2)^{th}$, $(r+3)^{rd}$, and $(r+4)^{th}$ terms in the expansion of $(1+x)^n$ be respectively a, b, c and d, then prove that $\frac{b^2-ac}{c^2-bd} = \frac{a(r+3)}{c(r+1)}$.

205. If a and b denote the sum of the coefficients in the expansions of $(1 - 3x + 3x^2)^n$ and $(1+x^2)^n$ respectively, then find relation between a and b.

206. Find the coefficient of x^{53} in the expression $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m}.2^m$.

207. Find the sum of the series $1^2.C_1 + 2^2.C_2 + 3^2.C_3 + \ldots + n^2.C_n$.

208. If $C_1, C_2, \ldots, C_n$ are binomial coefficients in the expansion of $(1+x)^n$, then prove that $1 - \left(\frac{1+x}{1+nx}\right)C_1 + \left(\frac{1+2x}{(1+nx)^2}\right)C_2 - \left(\frac{1+3x}{(1+nx)^3}\right)C_3 + \cdots + (-1)^n\left(\frac{1+nx}{(1+nx)^n}\right)C_n = 0$.

209. Evaluate $\sum_{r=0}^{n} \frac{3r+2}{r^3+6r^2++11r+6} .{}^nC_r$.

210. If $C_1, C_2, \ldots, C_n$ are binomial coefficients in the expansion of $(1+x)^n$ and $p + q = 1$, then prove that **(A)** $\sum_{r=0}^{n} r\ {}^nC_r p^r q^{n-r} = np$
(B) $\sum_{r=0}^{n} r^2\ {}^nC_r p^r q^{n-r} = n^2p^2 + npq$

211. Show that the greatest coefficient in the expansion of $\left(x + \frac{1}{x}\right)^{2n}$ is $[1.3.5 \ldots (2n-1)].\frac{2^n}{n!}$.

212. Prove that when $32^{(32)^{32}}$ divided by 7, the remainder is 4.

213. Find the remainder, when $(15^{23} + 23^{23})$ is divided by 19.

214. The term independent of x in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x > 0$ is k times the corresponding binomial coefficient. Find the value of k.

215. Find the coefficient of x^{-1} in $(1 + 3x^2 - x^4)\left(1 + \frac{1}{x}\right)^8$.

216. $(1+x)(1+x+x^2)(1+x+x^2+x^3)\ldots(1+x+x^2+\cdots+x^{100})$ when written in ascending powers of x, then the highest exponent of x is $\lambda(1010)$. Find the value of λ.

ANSWERS

1. DIY
2. DIY
3. $10x^4y + 20x^2y^3 + 2y^5$, 152
4. P.T.
5. P.T.
6. DIY
7. 0.951
8. **a.** $(1.01)^{1000000}$ **b.** $(1.1)^{10000}$ **c.** $(1.2)^{4000}$
9. P.T.
10. P.T.

11. P.T.
12. P.T.
13. P.T.
14.
a. ${}^{6}C_r(-1)^r x^{2(n-r)} y^r$
b. ${}^{99}C_r(-1)^r x^{198-r} y^r$
15. **a.** ${}^{18}C_{10} 3^6 x^3$
b. ${}^{n}C_{10} e^{x(n-20)}/2^n$
16. a. $-{}^{10}C_5$
b. $-{}^{7}C_3 \frac{3^4x^9}{7^3}, {}^{7}C_4 \frac{3^3x^{12}}{7^4}$
c. ${}^{12}C_6 \left(\frac{2ab}{3}\right)^6$
d. ${}^{2n}C_n(-1)^n x^n$
f. ${}^{6n}C_{3n} x^{3n}$
17. DIY
18. $\binom{23}{10}(xy)^{49/2}$
19. $-\binom{12}{7}3^{7/2}a^5b^7$
20. $n = 18$
21. $r = 13$
22. $n = 23$
23. P.T.
24. P.T.
25. DIY
26. P.T.
27. B
28. B
29. C
30. C
31. C
32. B
33. B
34. A
35. C
36. B
37. C
38. D
39. A
40. C
41. A
42. A
43. B
44. C
45. B
46. A
47. C
48. C
49. T_{10}
50. ${}^{11}C_4 2^7 3^4$
51. $m = 4$
52. No Term
53. P.T.
54. P.T.
55. T_5
56. $r = 14$
57. $n = 10$
58. $a = 1/2$
59. 171
60. x=$10^2, 10^{-\frac{1}{2}}$
61. $10^{\left(\frac{4\pm\sqrt{19}}{2}\right)}$
62. $10^{-4}, 10$
63. $\log_5 15$
64. $\lambda = -3/10$
65. $ab + 1 = 0$
66. $a = 9/7$
67. P.T.
68. P.T.
69. P.T.
70. $\sum_{n=31}^{40}({}^{n}C_5)$
71. n=12, r=3
72. ${}^{10}C_5$
73. P.T.
74. $n = 14,23$
75. $n = 3, r = 2$
76. P.T.
77. x=1, a=2, n=7
78. 6
79. 129
80. P.T.
81. P.T.
82. $a = 2b$
83. $a = b$
84. P.T.
85. 01
86. D
87. D
88. C
89. C
90. B
91. C
92. A
93. D
94. D
95. A
96. A
97. C
98. A
99. C
100. A
101. B
102. A
103. B
104. A
105. B
106. D
107. A
108. B
109. D
110. D
111. C
112. C
113. A
114. C
115. D
116. C
117. B
118. A
119. A
120. B
121. A
122. C
123. C
124. B
125. A
126. B
127. C
128. A
129. C
130. B
131. B
132. A
133. B
134. A
135. C
136. A
137. C
138. C
139. A
140. D
141. C
142. D
143. P.T.
144. P.T.
145. P.T.
146. P.T.
147. P.T.
148. P.T.
149. P.T.
150. 41
151. P.T.
152. P.T.
153. P.T.
154. A
155. C
156. C
157. D
158. C
159. A
160. A
161. D
162. A
163. C
164. B
165. C
166. A
167. A
168. C
169. C
170. B
171. B
172. B
173. B
174. B
175. C
176. (I)A (II) B (III) A
177. (I)B (II) D(III) A (IV) A (V)B
178. (I) C(II) C(III)C
179. (I) B(II) D(III)D
180. B,C
181. A,B,C,D
182. C,D
183. B,D
184. A,D
185. A,C
186. B,C,D
187. A,B
188. C,D
189. A,B,C
190. A,B,C
191. A,B,C
192. A,B,C,D
193. A,B,C,D
194. A,B,C,D
195. A,B,D
196. B,C,D
197. A,B,C,D
198. a=3,b=5,n=6
199. P.T.
200. P.T.
201. P.T.
202. P.T.
203. DIY*
204. P.T.
205. DIY
206. $-{}^{100}C_{53}$
207. $n(n+1)2^{n-2}$
208. P.T.
209. DIY
210. P.T.
211. P.T.
212. P.T.
213. 0
214. 1
215. 120
216. 5